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Numerical prediction of natural convection in a tall enclosure

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SUMMARY

The equations for natural convection flow in a tall cavity were solved for a super-critical Rayleigh number for which oscillations occur in the flow field and various solutions are possible. The objective was to compare various solution methods for this complex flow situation. The equations were solved using the finite element method with the Galerkin form of the method of weighted residuals with various time integration methods, time steps and grid spacings. The Euler time integration method is unsuitable for this problem because of its excessive dissipation. Fine grid distributions and small time steps were needed to predict accurate values of the average temperatures and velocities in the cavity, with even finer elements and time steps needed to accurately predict the amplitudes of the oscillations. An initially uniform temperature distribution lead to a uniformly oscillating skew-symmetric flow field. An initially random temperature field lead to a symmetry-breaking flow field that eventually reached the skew-symmetric flow field after a long time. Copyright © 2002 John Wiley & Sons, Ltd.

KEY WORDS: computation fluid dynamics; numerical accuracy; benchmarks; cavity flows; oscillatory flows; finite element method

INTRODUCTION

The natural convection flows in tall cavities develop into various complex time-dependent flow fields. This paper presents the solutions for a differentially heated 8:1 rectangular cavity filled with a Boussinesq fluid with $Ra = 340\,000$ and $Pr = 0.71$. This Rayleigh number is greater than the critical Rayleigh number where the flow begins to oscillate. Results are presented for two initial temperature profiles, a uniform initial temperature field and a random temperature distribution around a mean of zero. The uniform initial temperature profile results in a skew-symmetric solution while the random temperature distribution initially results in a second unstable mode which is symmetry breaking (not skew-symmetric), but which eventually reaches the skew-symmetric solution after a long time. Results for the uniform initial distribution are also presented for several different grid arrangements, time steps and calculational methods. The periods and amplitudes were evaluated for time periods that started after many uniform oscillations during which the amplitudes were essentially uniform.

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PROBLEM STATEMENT

The natural convection flow in a tall (8:1 height to width ratio) two-dimensional rectangular cavity was analysed numerically by solving the time-dependent Navier–Stokes equations with the energy equation using the Boussinesq approximation. The side walls had uniform temperatures with the left wall at $T = 0.5$ and the right wall at $T = -0.5$, while the top and bottom were perfectly insulated. The initial conditions were that the velocities were all zero and the temperature field was either uniform or a random distribution around a mean of zero. The problem set-up is described in detail by Christon *et al.* [1]. All variables used in this paper are non-dimensionalized as described by Christon *et al.* [1].

NUMERICAL METHOD

The problem was solved using the finite element method with the Galerkin form of the method of weighted residuals [2]. The transient term was modelled using either the forward/backward Euler predictor/corrector method or the Adams–Bashforth/trapezoidal rule. The elements were all 8-point quadrilaterals using the serendipity biquadratic interpolation functions. The algebraic equations were solved using Newton's method. Uniform and non-uniform grid distributions were used with either 21×101 nodes or 41×201 nodes in the horizontal and vertical directions, respectively. The non-uniform distributions had nodes bunched near the walls using a maximum (in the centre) to minimum (at the walls) element size ratio of 10 as described in the Nachos II manual [2].

Calculations were also attempted using the finite-difference technique with the SIMPLE technique to solve the equations with the advection term modelled using the hybrid method as implemented in PHOENICS version 1.4. However, since PHOENICS only uses single precision variables, the numerical inaccuracies with the large number of nodes overwhelmed the solution so that the results were obviously incorrect.

The calculations were done on a single processor 550 MHz Pentium III processor with 256 Mbytes of memory. Similar models listed in the specfp95 rating had floating point calculational rates of about 15.8 MFLOPs. The calculations for the 21×401 grid required 34 s per time step and 210 Mbytes memory.

RESULTS

For the supercritical Rayleigh number of 340 000 with a Prandtl number of 0.71, the velocities and temperatures begin to oscillate uniformly after the initial transient, which lasts until $t \approx 100$. The oscillation becomes fairly uniform at $t \approx 400$, but the amplitude of the oscillations continues to slowly grow until reaching a steady-state oscillation after $t \approx 800$. The flow for the symmetry-breaking scenario (scenario 3 in Table 1) also develops with an initial transient until $t \approx 100$ followed by the oscillations slowly growing until about $t \approx 1100$ and then decaying until the skew-symmetry flow field appears at $t \approx 2400$.

Results are presented for the various scenarios in Table 1. For scenario 4, the data are for the flow during the symmetry-breaking period as described below. The use of the Euler time step method always resulted in steady-state solutions due to its inherent dissipation; therefore, the Adams–Bashforth/trapezoidal rule was used for all the results presented here.

Table I. Point, wall and global data for various grid resolutions. Point data are for point 1, $x = 0.181$, $y = 7.370$.

	Grid: 21×101, non-uniform Uniform initial profile Time duration: steady			Grid: 41×201, uniform Uniform initial profile Averaging period: 20 cycles Starting time: 1270 Steps per period: 34 ($\Delta\tau = 0.1$)			Grid: 41×201, non-uniform Uniform initial profile Averaging period: 20 cycles Starting time: 1380 Steps per period: 34 ($\Delta\tau = 0.1$)			Grid: 41×201, non-uniform Non-uniform initial profile Averaging period: 20 cycles Starting time: 1100 Steps per period: 37 ($\Delta\tau = 0.1$)			Grid: 41×201, non-uniform Uniform initial profile Averaging period: 20 cycles Starting time: 1300 Steps per period: 137 ($\Delta\tau = 0.025$)		
Quantity	Average	Amplitude	Period	Average	Amplitude	Period	Average	Amplitude	Period	Average	Amplitude	Period	Average	Amplitude	Period
u	0.0511			0.0569	0.0576	3.42	0.0559	0.0504	3.43	0.0551	0.0432	3.68	0.0558	0.0493	3.42
v	0.4683			0.4621	0.0804	3.42	0.4624	0.0716	3.43	0.4622	0.0655	3.68	0.4624	0.0703	3.42
θ	0.2644			0.2654	0.0446	3.42	0.2655	0.0394	3.43	0.2656	0.0344	3.68	0.2655	0.0386	3.42
ε_{12}	0			0			0			0.000012	0.0600	3.68	0		3.42
ψ	−0.0751			−0.0729	0.0070	3.42	−0.0737	0.0062	3.43	−0.0739	0.0054	3.68	−0.0737	0.0061	3.42
ω	−2.6503			−2.312	1.144	3.42	−2.369	0.973	3.43	−2.406	0.844	3.68	−2.375	0.951	3.42
ΔP_{14}	0.01486			0.0059	0.0145	3.42	0.00550	0.0013	3.43	0.00537	0.012	3.68	0.00534	0.0021	3.42
ΔP_{51}	−0.5497			−0.5510	0.0153	3.42	0.5340	0.0180	3.43	−0.5340	0.0159	3.68	−0.5340	0.0019	3.42
ΔP_{35}	−0.5453			0.5513	0.0053	3.42	0.5333	0.0088	3.43	0.5332	0.0019	3.68	0.5337	0.0060	3.42
Nu ($x = 0$)	4.6381			4.6838	0.0087	3.42	4.5824	0.0066	3.43	4.5822	0.0082	3.68	4.5824	0.0065	3.42
Nu ($x = W$)	4.6381			4.6838	0.0087	3.42	4.5824	0.0066	3.43	4.5822	0.0082	3.68	4.5824	0.0065	3.42
$\hat{u}$	0.07009			0.2396	0.000035	3.42	0.2394	0.000031	3.43	0.2393	0.00004	3.68	0.2393	0.000031	3.42
$\hat{\omega}$	3.3763			2.8687	0.0033	3.42	3.0339	0.0030	3.43	3.0345	0.00099	3.68	3.0340	0.0029	3.42

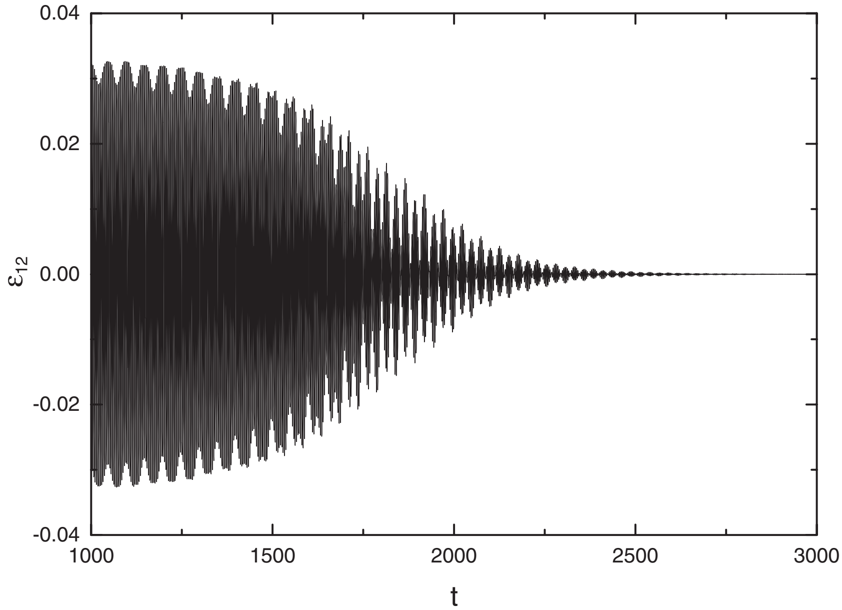

Figure 1. Decay of the temperature skew between points 1 and 2 for the symmetry-breaking scenario.

The first scenario with fewer nodal points resulted in a steady-state solution. The second scenario used finer grids distributed uniformly across the domain. The third, fourth and fifth scenarios used a non-uniform grid whose near-wall elements were about one-fifth the size of the corresponding grids in the uniform grid in scenario 2.

The fourth scenario began with a random temperature distribution having a mean of zero and a maximum variation of ± 0.5. The random initial temperature distribution resulted in a flow field that was initially symmetry breaking with oscillations that were somewhat slower than for the skew-symmetric scenario. However, after a long time, the flow eventually reached the skew-symmetric flow field as shown by the variation over time of the temperature skew, ε_{12}, plotted in Figure 1. Figure 1 shows a very large number of oscillations of the skew. The maximum amplitude initially increased until about $t = 1100$ and then slowly decreased to zero. The data in Table 1 for scenario 4 are for $t = 1100$–1173 when the amplitude was the largest. When the flow field eventually reached the skew-symmetric flow, the parameters were essentially the same as that for scenario 3. The average skew between the temperatures at points 1 and 2 listed in Table 1 for scenario 4 was nearly zero over a cycle, but the values varied considerably as shown by the amplitude of the oscillations plotted in Figure 1. The variation of the Nusselt number on the two sides of the cavity plotted in Figure 2 also shows how the flow field transformed from the symmetry-breaking flow field to the skew-symmetric flow field. The darker curve is for the cold side while the lighter dotted line is for the hot side with the figure showing many oscillations of both curves. The oscillations in the curves were initially out of phase but then became in phase after a long time as the skew-symmetric flow field developed.

The fifth scenario began with a uniform temperature distribution on the same non-uniform grid as for scenario 3 but used a much smaller time step with all of the other conditions

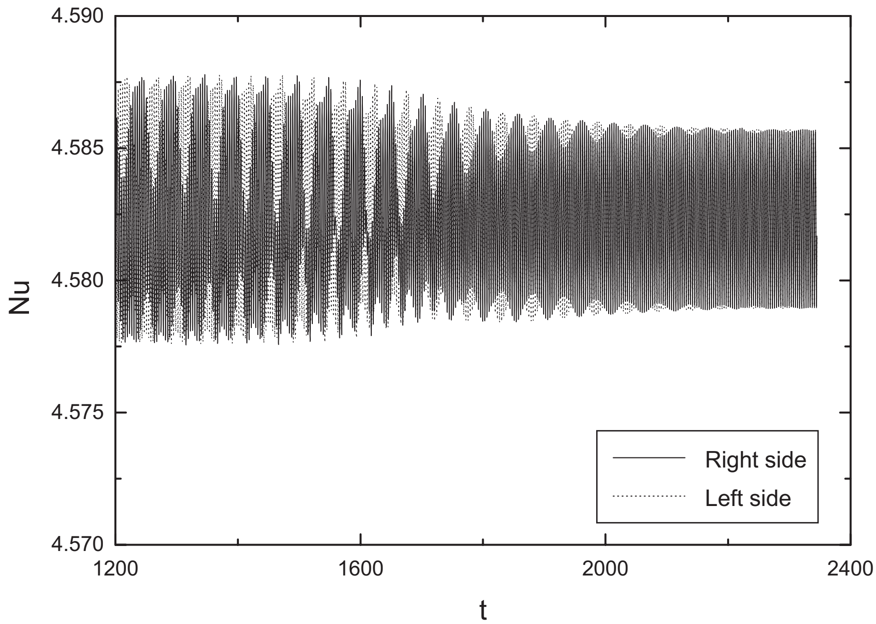

Figure 2. Variation of the Nusselt number for the symmetry-breaking scenario.

remaining the same as for scenario 3. The time step in scenario 5 allowed approximately 117 steps per oscillation while the time step in scenario 3 allowed approximately 34 steps per cycle which would normally be considered sufficient.

The average parameter values listed in Table 1 for the non-uniform grid, scenario 3, varied by about 2% from the values for the uniform grid, scenario 2, except for ΔP_{14} which was more sensitive to the grid size. Most of the amplitudes for the non-uniform grid scenario varied by less than 20% from the amplitudes for scenario 2 except for the pressure differences which were, in general, more sensitive to the grid. The parameter values for the finer grid, scenario 5, were then within 0.2% of the parameter values for scenario 3, again except for ΔP_{14}. Most of the amplitudes for the finer grid scenario were within 2% of the amplitudes for scenario 3 again, except for the pressure differences.

The variations of the temperature at point 1 over one cycle are plotted in Figure 3 for scenarios 2–5 in Table 1 that have oscillations. The data for scenario 4 are at $t \approx 1100$. The various curves illustrate that the time-step size and the grid distribution had a relatively small effect on the calculated temperatures.

CONCLUSIONS

The Galerkin form of the method of weighted residuals was used to solve the equations for natural convection in a tall two-dimensional cavity using the Adams–Bashforth/trapezoidal rule for the transient terms. Use of the Euler method for the transient term resulted in an incorrect steady-state result as did use of insufficient number of elements. For a 41×201

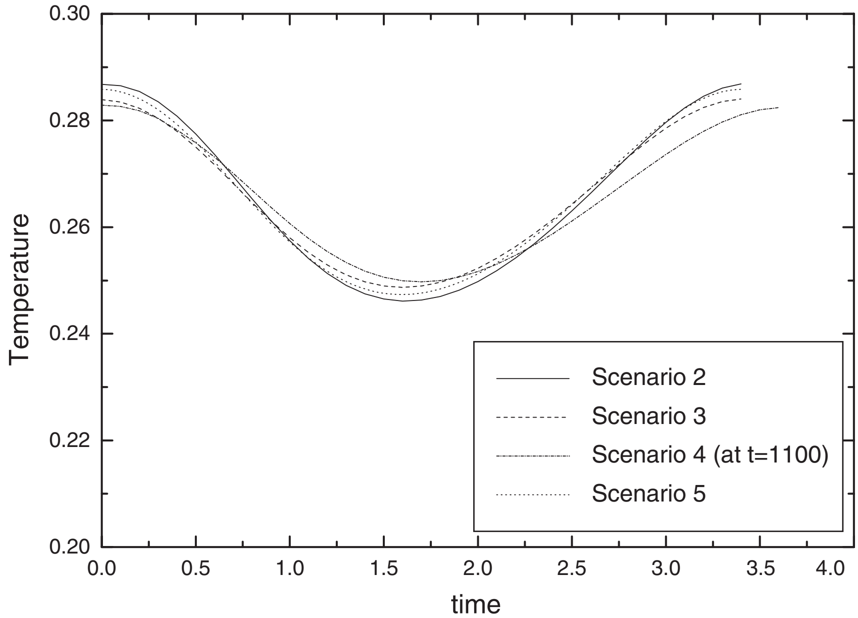

Figure 3. Temperature variations over one cycle for point 1 ($x=0.181$, $y=7.370$).

non-uniform grid, changing the time step from 0.1 to 0.025 had only a small effect on the results. More calculations are needed with more elements to determine if the grid distribution provides sufficiently accurate results. A random initial temperature profile initially resulted in a symmetry-breaking flow field that eventually developed into the standard skew-symmetric flow.

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